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Analysis of laminar heat transfer in micro-Poiseuille flow

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Abstract

The present study examines laminar forced convective heat transfer of a Newtonian fluid in a microchannel between two parallel plates analytically. The viscous dissipation effect, the velocity slip and the temperature jump at the wall are included in the analysis. Both hydrodynamically and thermally fully developed flow case is examined. Either the hot wall or the cold wall case is considered for the two different thermal boundary conditions, namely the constant heat flux (CHF) and the constant wall temperature (CWT). The interactive effects of the Brinkman number and the Knudsen number on the Nusselt numbers are analytically determined. Different definitions of the Brinkman number based on the definition of the dimensionless temperature are discussed. It is disclosed that for the cases studied here, singularities for the Brinkman number-dependence of the Nusselt number are observed and they are discussed in view of the energy balance.

Keywords: Microchannel; Parallel plate; MEMS; Flow physics; Viscous dissipation; Temperature jump; Velocity slip

1. Introduction

Research interest on flow and heat transfer phenomena in microchannels has increased substantially in recent years due to developments in the electronic industry, microfabrication technologies, biomedical engineering, etc.

Microscale fluid flow and heat transfer behavior differs greatly from that at macroscale. At macroscale, classical conservation equations are successfully coupled with the corresponding wall boundary conditions, usual no-slip for the hydrodynamic boundary condition and no-temperature-jump for the thermal boundary condition. These two boundary conditions are valid only if the fluid flow adjacent to the surface is in thermal equilibrium. However, they are not valid for rarefied gas flow at microscale. For this case, the gas no longer reaches the velocity or the temperature of the surface and therefore a slip condition for the velocity and a jump condition for the temperature should be adopted. The velocity slip and temperature jump are main effects of rarefaction. In fact, these are mathematical models taken from the kinetic theory of gases (for rarefied gases) to represent the results obtained experimentally in microchannel flows due to the similarity of the ratio between mean free path and the characteristic length of the problem. For a more detailed discussion on these, the readers are referred to the textbook by Gad-el-Hak [1].

Viscous dissipation is another parameter that should be taken into consideration at microscale. Viscous dissipation changes the temperature distributions by behaving like an energy source due to a considerable power generation induced by the shear stress, which, in the following, effects heat transfer rates. The merit of the effect of the viscous dissipation depends on whether the pipe is being cooled or heated.

A great deal of research have been undertaken to study the fundamentals of the transport phenomena in microchannels. Readers are referred to see the following recent excellent reviews related to transport phenomena in microchannels. Beskok and Karniadakis [2] defined four different flow regimes based on the value of the Knudsen number, Kn: Continuum flow (ordinary density levels) for $Kn \leqslant 0.001$; Slip-flow regime (slightly rarefied) for $0.001 \leqslant Kn \leqslant 0.1$; Transition regime (moderately rarefied) for $0.1 \leqslant Kn \leqslant 10$; Free-molecule flow (highly rarefied) for $10 \leqslant Kn \leqslant \infty$. Kn is the ratio of the gas mean free path, λ to the characteristic dimension in the flow field, H and, it determines the degree of rarefaction and the degree of the validity of the continuum approach. As Kn increases, rarefaction

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Nomenclature

A	cross-sectional area	m^2
Br	Brinkman number, Eq. (10)	
Br_q	modified Brinkman number, Eq. (13)	
c_p	specific heat at constant pressure	
F	tangential momentum accommodation coefficient	
F_t	thermal accommodation coefficient	
h	convective heat transfer coefficient	$W\,m^{-2}\,K^{-1}$
k	thermal conductivity	$W\,m^{-1}\,K^{-1}$
Kn	Knudsen number	
Nu	Nusselt number	
Pr	Prandtl number	
q_w	wall heat flux	$W\,m^{-2}$
Y	dimensionless vertical coordinate	
w	half distance between plates	m
T	temperature	K
u	velocity	$m\,s^{-1}$
z	axial direction	m

Greek symbols

α	thermal diffusivity	$m^2\,s^{-1}$
γ	specific heat ratio	
λ	molecular mean free path	
μ	dynamic viscosity	Pa s
ρ	density	$kg\,m^{-3}$
ν	kinematic viscosity	$m^2\,s^{-1}$
θ	dimensionless temperature, Eq. (8)	
θ_q	dimensionless temperature modified, Eq. (13)	
θ^*	dimensionless temperature, Eq. (16)	
θ_q^*	dimensionless temperature modified, Eq. (17)	

Subscripts

c	centerline
m	mean
s	fluid properties at the wall, singularity value
w	wall

effects become more important, and eventually the continuum approach breaks down. Ho and Tai [3] summarized discrepancies between microchannel flow behavior and macroscale Stokes flow theory. Palm [4], Sobhan and Garimella [5] and Obot [6] reviewed the experimental results in the existing literature for the convective heat transfer in microchannels. Rostami et al. [7,8] presented reviews for flow and heat transfer of liquids and gases in microchannels. Gad-el-Hak [1,9] broadly surveyed available methodologies to model and compute transport phenomena within microdevices. Guo and Li [10,11] reviewed and discussed the size effects on microscale single-phase fluid flow and heat transfer. In a recent study, Morini [12] presents an excellent review of the experimental data for the convective heat transfer in microchannels in the existing literature. He critically analyzed and compared the results in terms of the friction factor, laminar-to-turbulent transition and the Nusselt number.

Arkilic et al. [13] conducted an analytic and experimental investigation into gaseous flow with slight rarefaction through long microchannels. In the analytical analysis, they used a perturbation method to solve two-dimensional Navier–Stokes equations with first-order slip boundary conditions for a compressible gas flow in parallel plane microchannels. Gravesen et al. [14] explained deviations at the microscale from the macroscale in terms of wall slip and compressibility phenomena in microchannels. Gaseous flow in two-dimensional (2-D) micromachined channels for various Knudsen numbers in the range from 10^{-3} to 0.4 was experimentally and theoretically studied by Harley et al. [15]. They used nitrogen, helium, and argon gases. The measured friction factor was found to in good agreement with theoretical predictions assuming isothermal, locally fully developed, first-order, slip-flow. Barron et al. [16,17] extended the Graetz problem to slip-flow and developed simplified relationships to describe the effect of slip-flow on the convection heat transfer coefficient. For a given Graetz number, they concluded that the Nusselt number and the convection heat transfer rate increased as the Knudsen number increased. Ameel et al. [18] analytically treated the problem of laminar gas flow in microtubes with a constant heat flux boundary condition at the wall assuming a slip flow hydrodynamic condition and a temperature jump thermal condition at the wall. They disclosed that the fully developed Nusselt number decreased with Knudsen number. Yu and Ameel [19] analytically studied laminar slip-flow forced convection in rectangular microchannels by applying a modified generalized integral transform technique to solve the energy equation, assuming hydrodynamically fully developed flow. The effects of rarefaction and the fluid/wall interaction on the heat transfer were determined. The transition point at which the switch from heat transfer enhancement to reduction occurred was identified for different aspect ratios. In another study Yu and Ameel [20] found a universal finite Nusselt number for laminar slip flow heat transfer at the entrance of a conduit, which is valid for both isothermal and isoflux thermal boundary conditions and for any conduit geometry. Tso and Mahulikar [21–23] theoretically and experimentally studied the effect of the Brinkman number on convective heat transfer and flow transition in microchannels. Li et al. [24] theoretically analyzed the wall-adjacent layer effect on convective heat transfer for developed laminar flow of gas in micropassages. They derived analytical expressions for temperature profiles and heat transfer coefficients for fully developed laminar flow heat transfer of gases both in microtubes and plain microchannels. Tunc and Bayazitoglu [25] studied steady laminar hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions using the integral technique. They included temperature jump condition at the wall and viscous heating within the medium. Toh et al. [26] numerically investigated steady, laminar three-dimensional fluid flow and heat transfer phenomena inside heated microchannels using a finite volume method. They validated the numerical procedure by comparing the predicted local thermal resistances with available experimental data. It was found that the heat input lowers the frictional losses, particularly at lower Reynolds numbers. Xu et al. [27] theoretically analyzed and examined the effects of viscous dissipation in microchannel flows. They suggested a criterion to draw the limit of the significance of the viscous dissipation effects. Zhu et al. [28] conducted a theoretical analysis of the heat transfer characteristics in a parallel plate with microspacing using the boundary condition of the slip velocity and temperature jump. The influences of the Kn number, the tangential momentum accommodation coefficient, and the thermal accommodation coefficient on the temperature profile and the heat transfer characteristics were determined. Hadjiconstantinou [29] studied the effect of shear work at solid boundaries in small scale gaseous flows where slip effects were included. He illustrated the effect of shear work at the boundary on convective heat transfer solving the constant-wall-heat-flux problem in the slip-flow regime. Koo and Kleinstreuer [30,31] investigated the effects of viscous dissipation on the temperature field and ultimately on the friction factor using dimensional analysis and experimentally validated computer simulations. They showed that ignoring viscous dissipation could affect accurate flow simulations and measurements in microconduits. Hsieh et al. [32] presented an experimental and theoretical study of low Reynolds number flow of nitrogen in a microchannel. They concluded that using slip boundary conditions, one could well predict the mass flow rate as well as inlet/exit pressure drop and friction factor constant ratio for a three-dimensional physical system.

In a recent study, Aydın and Avcı [33] investigated the effect of the viscous dissipation on the heat transfer for a hydrodynamically developed but thermally developing Poiseuille flow.

The aim of the present study is to analytically investigate the slightly rarefied gas flow in a microchannel between two parallel plates. Both the constant wall temperature (CWT) and the constant heat flux (CHF) thermal boundary conditions are applied at the wall. Either the hot wall (the fluid is heated) case or the cold wall (the fluid is cooled) case is considered. The slip flow regime ($0.001 \leqslant Kn \leqslant 0.1$) is assumed to exist. To account for non-continuum effects in the slip flow regime, the governing equations are solved in conjunction with the velocity slip and temperature jump. The combined effects of the Brinkman number and the Knudsen number on the temperature profile and, in the following, the Nusselt number are determined for the above conditions.

2. Analysis

Here, both hydrodynamically and thermally fully developed flow case is considered. Steady, laminar flow having constant properties (i.e. the thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature) is considered. The axial heat conduction in the fluid and in the wall is assumed to be negligible. For the laminar flow regime in microchannels, the Reynolds number is lower than 2000. The common gases usually have a Prandtl number in the range between 0.6–0.9. Since the Peclet number, $Pe(= Re\,Pr)$ is greater than 100 for $Re > 150$, it is reasonable to neglect the axial conduction in the fluid.

In this study, the usual continuum approach is coupled with the two main characteristics of the microscale phenomena, the velocity slip and the temperature jump, following the Ref. [1]. The velocity slip is prescribed as:

$$u_s = -\frac{2-F}{F}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=w} \tag{1}$$

where u_s is the slip velocity, λ the molecular mean free path, and F is the tangential momentum accommodation coefficient, and the temperature jump is defined as [1]:

$$T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left.\frac{\partial T}{\partial y}\right|_{y=w} \tag{2}$$

where T_s is the temperature of the gas at the wall, T_w the wall temperature, and F_t is the thermal accommodation coefficient, which depends on the gas and surface material. Particularly for air, it assumes typical values near unity [18]. For the rest of the analysis, F and F_t will be shown by F.

The fully developed velocity profile taking the slip flow condition at the wall into consideration is derived from the momentum equation as:

$$\frac{u}{u_m} = \frac{3}{2}\left[\frac{1-(y/w)^2+4Kn}{1+6Kn}\right] \tag{3}$$

where u_m is the mean velocity and Kn is the Knudsen number, $Kn = \lambda/2w$.

The conservation of energy including the effect of the viscous dissipation requires

$$u\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 \tag{4}$$

where the second term in the right-hand side is the viscous dissipation term.

The axisymmetry at the center, $y = 0$ leads to the thermal boundary condition as follows:

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \tag{5}$$

Thermal boundary conditions of constant wall heat flux (CHF) and constant wall temperature (CWT) at wall are considered and they are separately treated in the following:

2.1. Constant heat flux

The constant heat flux at wall states that

$$k\left.\frac{\partial T}{\partial y}\right|_{y=w} = q_w \tag{6}$$

where q_w is positive when its direction is to the fluid (the hot wall), otherwise it is negative (the cold wall).

For the uniform wall heat flux case, the first term in the left-side of Eq. (4) is

$$\frac{\partial T}{\partial z} = \frac{\mathrm{d}T_w}{\mathrm{d}z} = \frac{\mathrm{d}T_s}{\mathrm{d}z} \tag{7}$$

By introducing the following non-dimensional quantities

$$Y=\frac{y}{w},\qquad \theta=\frac{T_s-T}{T_s-T_c} \tag{8}$$

Eq. (4) can be written as

$$\frac{d^2\theta}{dY^2}=a\frac{3}{2}\left[\frac{1-Y^2+4Kn}{1+6Kn}\right]+9Br\frac{Y^2}{(1+6Kn)^2} \tag{9}$$

where $a=-\frac{u_m w^2}{\alpha(T_s-T_c)}\frac{dT_s}{dz}$ and Br is the Brinkman number given as:

$$Br=\frac{\mu u_m^2}{k(T_s-T_c)} \tag{10}$$

For the solution of the dimensionless energy transport equation given in Eq. (9), the dimensionless boundary conditions are given as follows:

$$\theta=1,\qquad \left.\frac{\partial\theta}{\partial Y}\right|_{Y=0}=0\quad \text{at } Y=0$$
$$\theta=0\quad \text{at } Y=1 \tag{11}$$

The solution of Eq. (9) under the thermal boundary conditions given in Eq. (11) is

$$\begin{aligned}\theta(Y)&=\frac{T_s-T}{T_s-T_c}\\&=\frac{1}{5+24Kn}\biggl[5-6Y^2+Y^4+24Kn\bigl(1-Y^2\bigr)\\&\quad\times\frac{Br}{(1+6Kn)^2}\left(\frac{9}{2}Y^4-\frac{9}{2}Y^2+18Kn\bigl(Y^4-Y^2\bigr)\right)\biggr]\end{aligned} \tag{12}$$

As it is usual in the existing literature, we can also use the modified Brinkman number which is in the following:

$$Br_q=\frac{\mu u_m^2}{wq_w} \tag{13}$$

In terms of the modified Brinkman number, the temperature distribution is obtained as:

$$\begin{aligned}\theta_q&=\frac{T-T_s}{\frac{q_w w}{k}}\\&=\left[\frac{1}{1+6Kn}+\frac{3Br_q}{(1+6Kn)^3}\right]\\&\quad\times\left[-\frac{5}{8}+\frac{3}{4}Y^2-\frac{1}{8}Y^4+3Kn\bigl(Y^2-1\bigr)\right]\\&\quad-\frac{3Br_q}{4(1+6Kn)^2}\bigl(Y^4-1\bigr)\end{aligned} \tag{14}$$

Eqs. (12) and (14) which are in terms of T_s (note again T_s is the temperature of the fluid at the wall) can be transformed into the equations in terms of T_w, the wall temperature using the following conversion formula:

$$\frac{T_s-T_w}{\frac{q_w w}{k}}=-\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}$$
$$\frac{T_s-T_w}{T_s-T_c}=\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=1} \tag{15}$$

Then Eqs. (12) and (14) become, respectively,

$$\begin{aligned}\theta^*(Y)&=\frac{T_w-T}{T_s-T_c}\\&=\frac{1}{5+24Kn}\biggl[\biggl[5-6Y^2+Y^4+24Kn\bigl(1-Y^2\bigr)\\&\quad+\frac{Br}{(1+6Kn)^2}\left(\frac{9}{2}Y^4-\frac{9}{2}Y^2+18Kn\bigl(Y^4-Y^2\bigr)\right)\biggr]\\&\quad-\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-8-48Kn+Br\frac{9+36Kn}{(1+6Kn)^2}\right]\biggr]\end{aligned} \tag{16}$$

and

$$\begin{aligned}\theta_q^*&=\frac{T-T_w}{\frac{q_w w}{k}}\\&=\left[\frac{1}{1+6Kn}+\frac{3Br_q}{(1+6Kn)^3}\right]\\&\quad\times\left[-\frac{5}{8}+\frac{3}{4}Y^2-\frac{1}{8}Y^4+3Kn\bigl(Y^2-1\bigr)\right]\\&\quad-\frac{3Br_q}{4(1+6Kn)^2}\bigl(Y^4-1\bigr)-\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\end{aligned} \tag{17}$$

For a fully developed flow, it is usual to utilize the mean fluid temperature, T_m, rather than the center line temperature when defining the Nusselt number. This mean or bulk temperature is defined as:

$$T_m=\frac{\int\rho uT\,dA}{\int\rho u\,dA} \tag{18}$$

The dimensionless mean temperature in terms of Br is obtained as

$$\begin{aligned}\theta_m^*&=\frac{T_w-T_m}{T_c-T_s}\\&=\frac{2}{3}+\frac{Br}{35(1+6Kn)^3}-\frac{9Br}{35(1+6Kn)^2}\\&\quad+\frac{2(8+33Br)}{105(1+6Kn)}-\frac{22(1+12Br)}{105(5+48Kn)}\\&\quad-\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-8-48Kn+Br\frac{9+36Kn}{(1+6Kn)^2}\right]\end{aligned} \tag{19}$$

while, in terms of

$$\begin{aligned}\theta_{q,m}^*&=\frac{T_m-T_w}{\frac{q_w r_0}{k}}\\&=-\frac{1}{3}\left(1+\frac{12\gamma}{\gamma+1}\frac{Kn}{Pr}\right)-\frac{2Br_q}{35(1+6Kn)^4}-\frac{11Br_q}{35(1+6Kn)^3}\\&\quad-\frac{2(1+21Br_q)}{105(1+6Kn)^2}-\frac{2}{15(1+6Kn)}\end{aligned} \tag{20}$$

2.2. Constant wall temperature

When the constant temperature is considered, since $\frac{dT_s}{dz}=0$, the first term in the left-side of Eq. (4) is

$$\frac{\partial T}{\partial z}=\left(\frac{T-T_s}{T_c-T_s}\right)\frac{dT_c}{dz} \tag{21}$$

Substituting this result into Eq. (4) and introducing the dimensionless quantities given in Eq. (8) gives the following dimensionless equation for CWT case:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}Y^2} = a\frac{3}{2}\left[\frac{1-Y^2+4Kn}{1+6Kn}\right]\theta + 9Br\frac{Y^2}{(1+6Kn)^2} \tag{22}$$

where $a = -\frac{u_m w^2}{\alpha(T_s-T_c)}\frac{\mathrm{d}T_c}{\mathrm{d}z}$ and Br is the Brinkman number. The boundary conditions given in Eq. (11) are also valid for this case. Actually, no simple closed form solution can be obtained for this equation. However, the variation of θ can be quite easily obtained to any required degree of accuracy by using an iterative procedure [34]. The temperature profile for the CHF case is used as the first approximation and Eq. (22) is then integrated to obtain θ. This iterative procedure is repeated until an acceptable convergence is reached.

The forced convective heat transfer coefficient is given as follows:

$$h = \frac{k\frac{\partial T}{\partial y}\big|_{y=w}}{T_w - T_m} \tag{23}$$

which is obtained from the following Nusselt number expressions based on θ^* and θ_q^*, respectively:

$$Nu = \frac{h(2w)}{k} = -\frac{2\frac{\partial\theta^*}{\partial Y}\big|_{Y=1}}{\theta_m^*} \tag{24}$$

$$Nu = \frac{h(2w)}{k} = -\frac{2}{\theta_{q,m}^*} \tag{25}$$

After performing necessary substitutions, for the CHF case, we obtain:

$$\begin{aligned} Nu = {} & \frac{2}{5+24Kn}\left[8+48Kn - Br\frac{9+36Kn}{(1+6Kn)^2}\right] \\ & \times \left[\frac{2}{3} + \frac{Br}{35(1+6Kn)^3} - \frac{9Br}{35(1+6Kn)^2} \right. \\ & + \frac{2(8+33Br)}{105(1+6Kn)} - \frac{22(1+12Br)}{105(5+48Kn)} \\ & \left. - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-8-48Kn+Br\frac{9+36Kn}{(1+6Kn)^2}\right]\right]^{-1} \end{aligned} \tag{26}$$

In terms of the modified Brinkman number, Br_q, the Nusselt number receives the following form:

$$\begin{aligned} Nu = 2\Bigg[& \frac{1}{3}\left(1+\frac{12\gamma}{\gamma+1}\frac{Kn}{Pr}\right) + \frac{2Br_q}{35(1+6Kn)^4} + \frac{11Br_q}{35(1+6Kn)^3} \\ & + \frac{2(1+21Br_q)}{105(1+6Kn)^2} + \frac{2}{15(1+6Kn)}\Bigg]^{-1} \end{aligned} \tag{27}$$

3. Results and discussion

The Brinkman number and the Knudsen number are the main parameters governing heat and fluid flow in a microchannel between two parallel plates. Here, their interactive effects on transport phenomena for both hydrodynamically and thermally fully developed flow in a microchannel are analyzed. And, $Br = 0$ or $Br_q = 0$ represents the case without the effect of the viscous dissipation. As stated earlier, two different thermal boundary conditions at the channel walls have been examined in this study, namely constant heat flux (CHF) and constant wall temperature (CWT). They are treated separately in the following.

In order to validate the method of analysis developed here, we compare some limiting results against those available in the existing literature, mainly by those of Hadjiconstantinou [29]. Table 1 represents comparison of our results with those in terms of the Nusselt numbers for different Knudsen and Brinkman numbers. As seen, an excellent agreement is observed. Note that Hadjiconstantinou [29] covered a narrow range of the Brinkman number range of $-0.25 \leqslant Br \leqslant 0.2$ for only the constant heat flux case at the channel walls.

Fig. 1 illustrates the variation of the Nusselt number with the Knudsen number for different Brinkman numbers for the CHF case. As seen, an increase at *Kn* decreases *Nu* due to the temperature jump at the wall. Viscous dissipation, as an energy source, severely distorts the temperature profile. Remember positive values of *Br* correspond to the wall heating or the hot wall (heat is being supplied across the walls into the fluid) case ($T_w > T_c$), while the opposite is true for negative values of *Br*. In the absence of viscous dissipation the solution is independent of whether there is wall heating or cooling. However, viscous dissipation always contributes to internal heating of the fluid, hence the solution will differ according to the process taking place. *Nu* decreases with increasing *Br* for the hot wall (i.e. the

Table 1
The fully developed Nusselt number values, $Pr = 0.71$ for the CHF case

Kn	Present		Ref. [25]	Present	Ref. [25]
	$Br = 0.0$	$Br_q = 0.0$	$Br = 0.0$	$Br = 0.01$	$Br = 0.01$
0.00	4.118	4.118	4.118	4.078	4.086
0.02	3.750	3.750	3.750	3.725	3.729
0.04	3.421	3.421	3.421	3.405	3.405
0.06	3.131	3.131	3.131	3.120	3.119
0.08	2.878	2.878	2.878	2.869	2.869
0.10	2.657	2.657	2.657	2.652	2.651

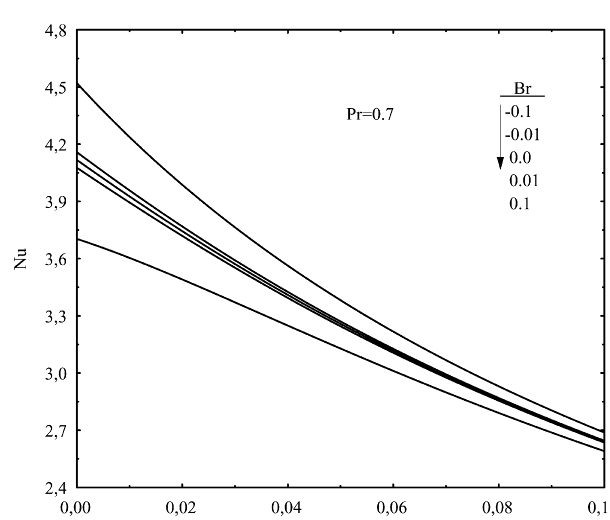

Fig. 1. The variation of the Nusselt number with the Knudsen number at different values of the Brinkman number for the CHF case.

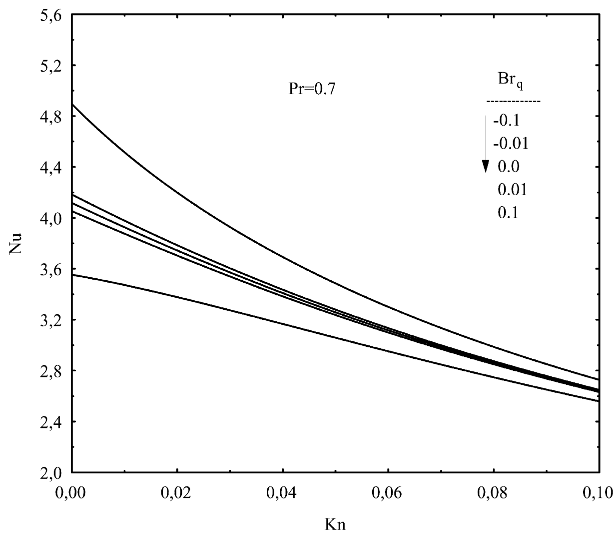

Fig. 2. The variation of the Nusselt number with the Knudsen number at different values of the modified Brinkman number for the CHF case.

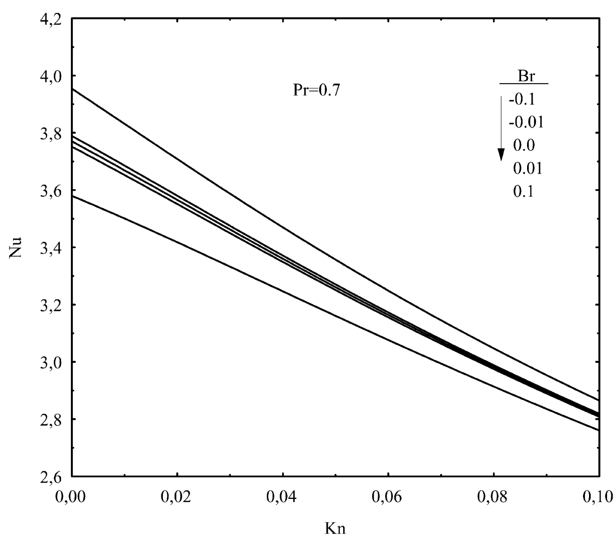

Fig. 3. The variation of the Nusselt number with the Knudsen number at different values of the Brinkman number for the CWT case.

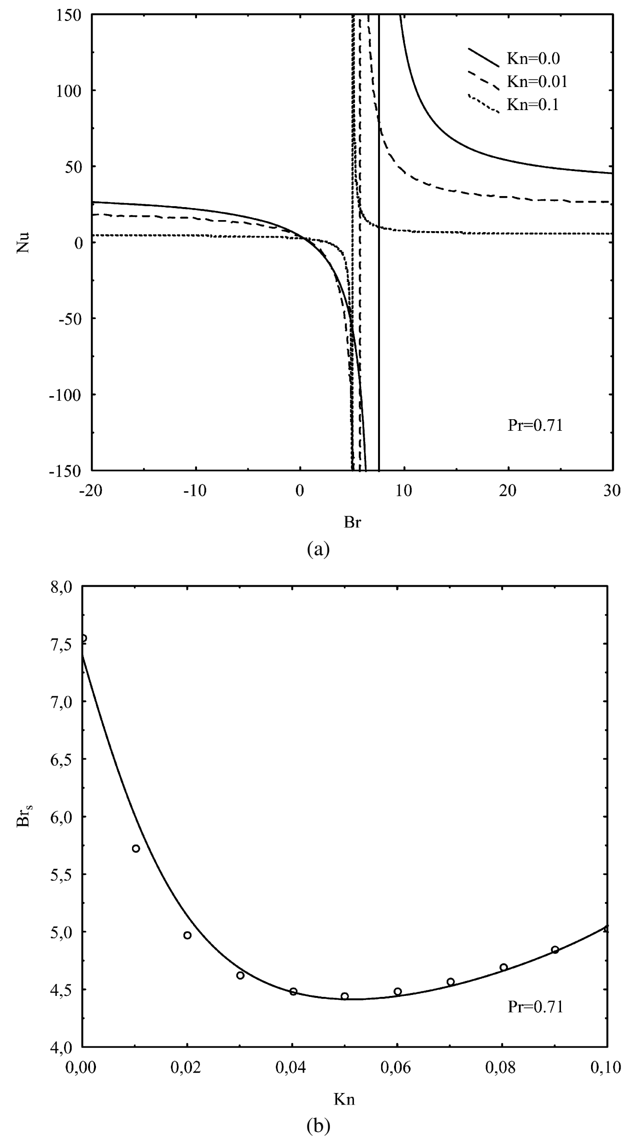

Fig. 4. (a) The influence of *Br* on *Nu* at various *Kn* for the CHF case. (b) The variation of Br_s with Knudsen number for the CHF case.

wall heating case). For this case, the wall temperature is greater than that of the bulk fluid. Viscous dissipation increases the bulk fluid temperature especially near the wall since the highest shear rate occurs in this region. Hence, it decreases the temperature difference between the wall and the bulk fluid, which is the main driving mechanism for the heat transfer from wall to fluid. However, for the cold wall (i.e. the wall cooling case), the viscous dissipation increases the temperature differences between the wall and the bulk fluid by increasing the fluid temperature more. As seen from Fig. 1, increasing *Br* in the negative direction increases *Nu*. This situation is an indication of the aiding effect of the viscous dissipation on heat transfer for the wall cooling case. As seen from Fig. 1, the behavior of *Nu* versus *Kn* for lower values of the Brinkman number, either in the case of wall heating ($Br = 0.01$) or in the case of the wall cooling ($Br = -0.01$) is very similar to that of $Br = 0$. In addition, as observed from Fig. 1, *Br* is more effective on *Nu* for lower values of *Kn* than for higher values of *Kn*.

The standard way of making temperature dimensionless based on Eq. (8) is not appropriate for the situation of imposed heat flux because the temperature scale $\Delta T = T_s - T_c$ varies with relevant parameters and may cause to a misinterpretation of the corresponding variation of T. In fact, for a given q_w, ΔT is the unknown of the problem and it is more convenient to define a fixed temperature scale that we take as $q_w w/k$. Fig. 2 shows the variation of the Nusselt number with the Knudsen number for different values of the modified Brinkman number, Br_q. The behavior of *Nu* versus *Kn* for different Br_q is very similar to for different *Br* (Fig. 1). As expected, increasing dissipation increases the bulk temperature of the fluid due to internal heating of the fluid. For the wall heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the bulk fluid, which is followed

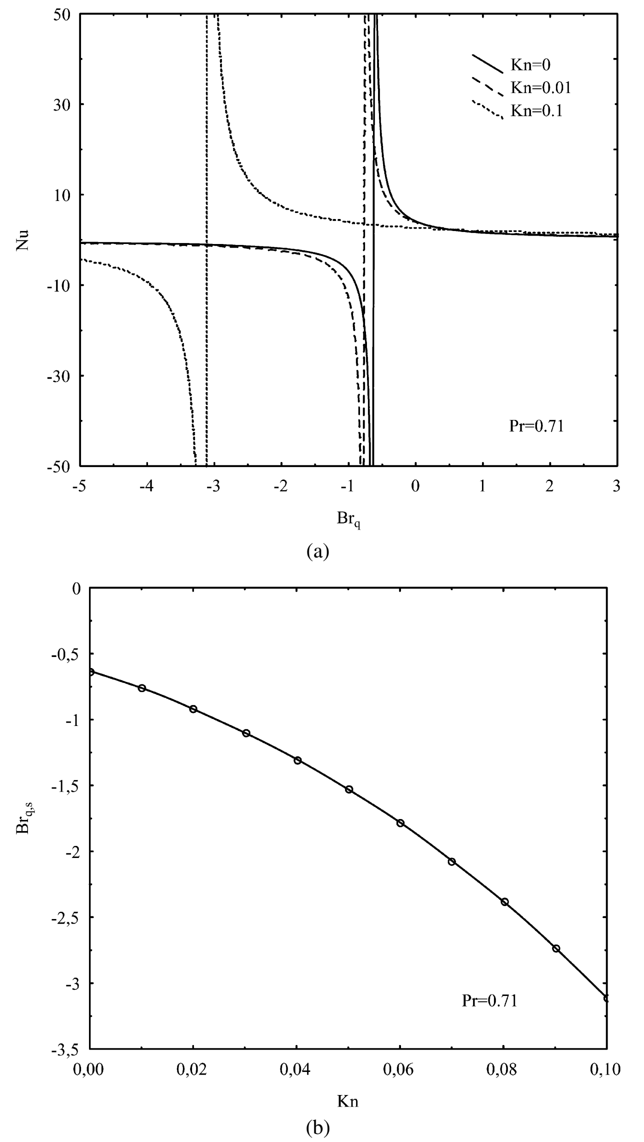

Fig. 5. (a) The influence of Br_q, on Nu at various Kn for the CHF case. (b) The variation of $Br_{q,s}$ with Knudsen number for the CHF case.

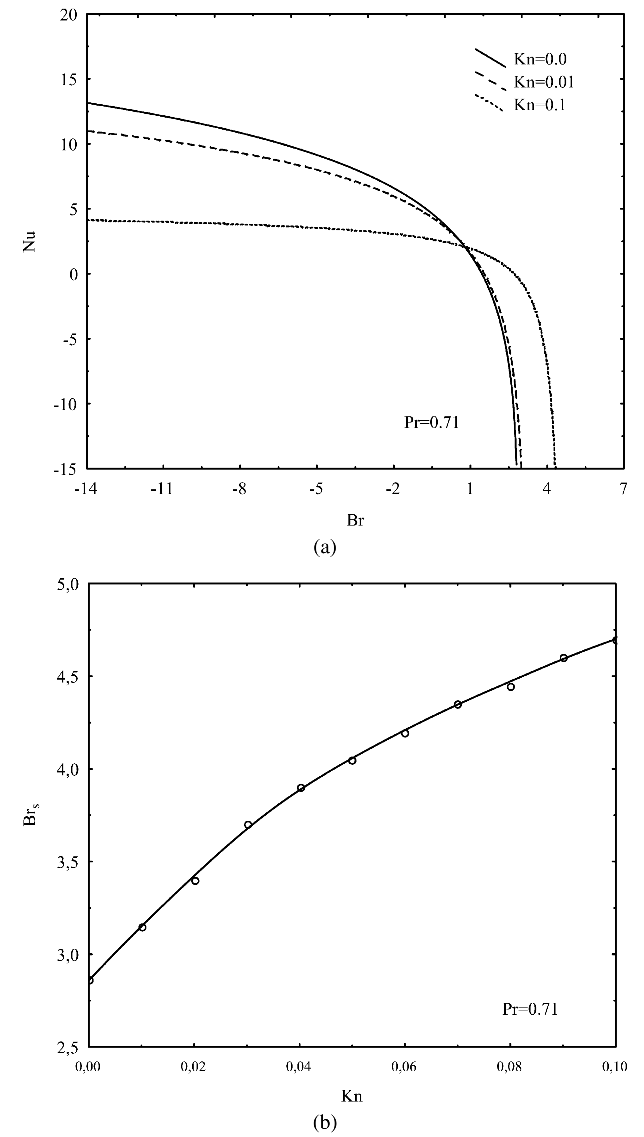

Fig. 6. (a) The influence of Br, on Nu at various Kn for the CWT case. (b) The variation of Br_s with Knudsen number for the CWT case.

with a decrease in heat transfer. When wall cooling is applied, due to the internal heating effect of the viscous dissipation on the fluid temperature profile, temperature difference is increased with the increasing Br_q.

Fig. 3 illustrates the effect of Kn on Nu for different values of Br at the CWT case. Similar behaviors have been observed to those obtained for the CHF case. As seen, for the same value of Br, Nu values are found to be lower than those for the CHF case.

In this study, as an original attempt, the Brinkman number range for the above cases considered are extended. Fig. 4(a) shows the influence of Br on Nu for various Kn. As shown, a singularity is observed at Br for each Kn. Actually, this is an expected result, when Eq. (26) is closely examined. For the hot wall case, at $Kn = 0$, with the increasing value of Br, Nu decreases in the range of $0 < Br < 7.56$. This is due to decreasing temperature difference between the wall and the bulk fluid. At $Br = 7.56$, the heat supplied by the wall into the fluid is balanced with the internal heat generation due to the viscous heating. For $Br > 7.56$, the internally generated heat by the viscous dissipation overcomes the heat supplied by the wall. When $Br \to +\infty$, Nu reaches an asymptotic value. When wall cooling ($Br < 0$) is applied to reduce the bulk temperature of the fluid, as explained earlier, the amount of viscous dissipation may change the overall heat balance. With increasing value of Br in negative direction, the Nusselt number reaches an asymptotic value. As shown Fig. 4(b), the singular value of the Brinkman number, Br_s decreases with increasing Knudsen number. Using the modified Brinkman number for the CHF case, the variation of Nu with Br_q for different Kn is shown in Fig. 5(a). Again, singularities are observed. As noticed, when Br_q goes to infinity for either wall heating or wall cooling case, the Nusselt number reaches the same asymptotic value, $Nu = 0$. This is due to fact that the heat generated internally by viscous dissipation processes will balance the effect of wall heating or cooling, reaching a thermal equilibrium condition. Fig. 5(b) shows the variation of the singular value of the modified Brinkman number, $Br_{q,s}$ with the increasing Knudsen number. As seen, $Br_{q,s}$ decreases with Kn.

For the CWT case, Fig. 6(a) illustrates the variation of Nu with Br. The behavior observed can be explained similarly to that for the CHF condition. Again, singularities are observed. And, the variation of the singular value of the Brinkman number, Br_s with Kn is given in Fig. 6(b). An increase at Kn results in increasing Br_s.

4. Conclusions

Here, we studied heat transfer for laminar rarefied gas flow in microchannels between two parallel plates with equal temperatures or heat fluxes. The analysis includes the influence of the viscous dissipation in addition to the slip velocity and temperature jump prescriptions at the wall. The interactive effects of the Brinkman number and Knudsen number on the Nusselt number have been studied. Two types of wall thermal boundary condition have been considered, namely: constant heat flux (CHF) and constant wall temperature (CWT). Both the hot wall (wall heating) and the cold wall (wall cooling) cases are treated. The variation of the Nusselt number with the Brinkman or the modified Brinkman number presented some singularities. It is disclosed that the values of these singularities are found to be affected by the value of the Knudsen number. The origins of these singularities have been explained in view of the energy balance.

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